Name:

Precalculus (Math 1045)
Practice Final Examination

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The examination contains ten problems which are worth 10 points each, and two additional extra credit problems worth 10 points each.

No books, notes, calculators, or other electronic devices are permitted.

Using trigonometric identities and the following table, it is possible to accurately compute the sine, cosine, tangent, cotangent, secant, and cosecant of the angles 0°, 15°, 18°, 30°, 36°, 45°, 60°, 72°, 75°, 90°, and many others.

θ	$0°$	$45°$	$60°$	$72°$
$\cos\theta$	1	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{5}-1}{2}$

Prob 1	Prob 2	Prob 3	Prob 4	Prob 5	Prob 6	Prob 7	Prob 8	Prob 9	Prob 10	Total Score

Problem 1. (Sets and Functions)
Let $A = \{1, 3, 5, 7, 9\}$, $B = \{3, 4, 5, 6, 7\}$, $C = [0, 6]$, and $D = (4, 9]$. Write each set as a union of disjoint intervals.

(a) $E = C \cup D$

(b) $F = D \setminus C$

(c) $G = E \setminus (A \cap B)$

(d) $H = G \cup B$

(e) $I = H \cap C$

Define $f : B \to \mathbb{R}$ by $f(x) = \dfrac{x^2 - 7}{2}$.

(f) Find the domain and range of f.

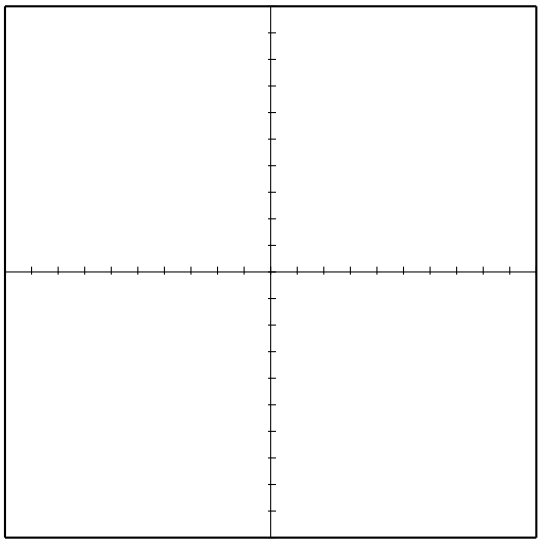
(g) Is f injective?

(h) Is f surjective?

Problem 2. (Quadratic Functions)
Analyze the polynomial function

$$f(x) = 2x^2 + x - 6$$

by filling in the table below, and use this information to sketch the graph.



Equation: $y = 2x^2 + x - 6$

Normal Form:

Shifted Form:

a: **b:** **c:** **h:** **k:**

Discriminant:

Zeros:

y-intercept:

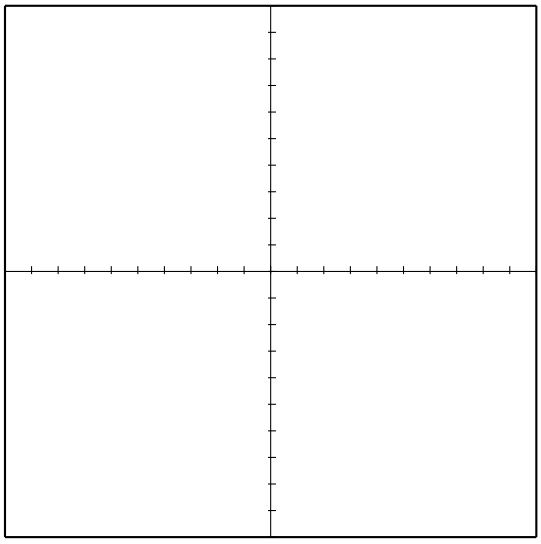
x-intercept(s):

Vertex:

Problem 3. (Rational Functions)
Analyze the rational function

$$f(x) = \frac{x^2 + x - 6}{x - 3}$$

by filling in the table below, and use this information to sketch the graph.



Equation: $y = \dfrac{x^2 + x - 6}{x - 3}$

Degree:

Zeros:

Poles:

y-intercept:

x-intercepts:

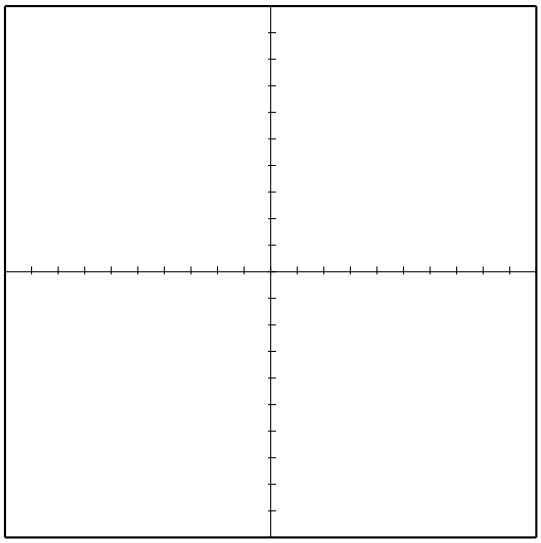
Vertical Asymptotes:

Polynomial Asymptote:

Problem 4. (Polynomial Functions)
Analyze the polynomial function

$$f(x) = x^4 + x^3 - 7x^2 - x + 6$$

by filling in the table below, and use this information to sketch the graph.



Polynomial: $y = x^4 + x^3 - 7x^2 - x + 6$

Degree:

Leading Coefficient:

Constant Coefficient:

Zeros:

y-intercept:

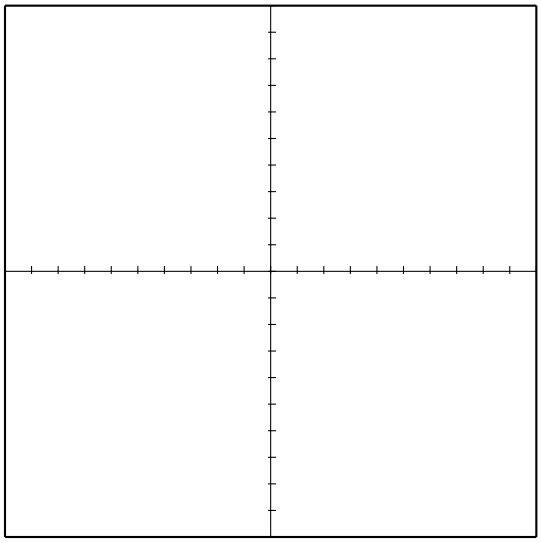
x-intercepts:

Shape:

Problem 5. (Hyperbolic Equations)
Consider the equation of an hyperbola with horizontal focal axis

$$9x^2 = 4y^2 + 36.$$

Put the equation in standard form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1.$. Identify the constants a, b, c, h, and k, where c denotes the distance from the center to a focus. Identify the center, vertices, covertices, foci, and asymptotes. Graph the hyperbola, including all of these features.



Standard Form:

a: **b:** **c:** **h:** **k:**

Center:

Vertices:

Covertices:

Foci:

Asymptotes:

Problem 6. Let

$$f(x) = x^8 - 8x^4 - 9.$$

Find all real and complex zeros of f.

Problem 7. (Solving Triangles)
Let a, b, c be the lengths of the sides of a triangle with corresponding opposite angles α, β, and γ. If $a = 4$, $b = 7$, and $\gamma = 60°$, find c.

Problem 8. (Vectors and Complex Numbers) Let $\vec{v} = \langle 2, 1\rangle$ and $\vec{w} = \langle -1, y\rangle$. Find y so that the angle between $\vec{v}$ and $\vec{w}$ is $60°$.

Problem 9. (Trigonometric Functions)

(a) Find all values for x in the range $[0°, 360°)$ which satisfy the equation

$$4\cos^2 x + 2\cos x = \sin^2 x + \cos^2 x.$$

(b) Find an algebraic expression for the function

$$f(x) = \sec\left(\arcsin\left(\frac{x}{3}\right)\right).$$

Problem 10. (Exponential and Logarithmic Functions)
Solve for x.

(a) $49^{(2x+1)} = \frac{1}{343^{(x-5)}}$

(b) $\log_{(x+1)}(5x+3) + \log_{(x+1)} x = 3$